NAG Fortran Library Routine Document

G13CEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

For a bivariate time series, G13CEF calculates the cross amplitude spectrum and squared coherency, together with lower and upper bounds from the univariate and bivariate (cross) spectra.

2 Specification

```
 SUBROUTINE G13CEF(XG, YG, XYRG, XYIG, NG, STATS, CA, CALW, CAUP, T, SC,
1                  SCLW, SCUP, IFAIL)
 INTEGER           NG, IFAIL
 real              XG(NG), YG(NG), XYRG(NG), XYIG(NG), STATS(4), CA(NG),
1                  CALW(NG), CAUP(NG), T, SC(NG), SCLW(NG), SCUP(NG)
```

3 Description

Estimates of the cross amplitude spectrum $A(\omega)$ and squared coherency $W(\omega)$ are calculated for each frequency ω as

$$A(\omega) = |f_{xy}(\omega)| = \sqrt{cf(\omega)^2 + qf(\omega)^2} \quad \text{and}$$

$$W(\omega) = \frac{|f_{xy}(\omega)|^2}{f_{xx}(\omega) f_{yy}(\omega)},$$

where

$cf(\omega)$ and $qf(\omega)$ are the co-spectrum and quadrature spectrum estimates between the series, i.e., the real and imaginary parts of the cross spectrum $f_{xy}(\omega)$ as obtained using G13CCF or G13CDF;

$f_{xx}(\omega)$ and $f_{yy}(\omega)$ are the univariate spectrum estimates for the two series as obtained using G13CAF or G13CBF.

The same type and amount of smoothing should be used for these estimates, and this is specified by the degrees of freedom and bandwidth values which are passed from the calls of G13CAF or G13CBF.

Upper and lower 95% confidence limits for the cross amplitude are given approximately by

$$A(\omega)\left[1 \pm (1.96/\sqrt{d})\sqrt{W(\omega)^{-1} + 1}\right],$$

except that a negative lower limit is reset to 0.0, in which case the approximation is rather poor. The user is therefore particularly recommended to compare the coherency estimate $W(\omega)$ with the critical value T derived from the upper 5% point of the F-distribution on $(2, d-2)$ degrees of freedom:

$$T = \frac{2F}{d - 2 + 2F},$$

where d is the degrees of freedom associated with the univariate spectrum estimates. The value of T is returned by the routine.

The hypothesis that the series are unrelated at frequency ω, i.e., that both the true cross amplitude and coherency are zero, may be rejected at the 5% level if $W(\omega) > T$. Tests at two frequencies separated by more than the bandwidth may be taken to be independent.

The confidence limits on $A(\omega)$ are strictly appropriate only at frequencies for which the coherency is significant. The same applies to the confidence limits on $W(\omega)$ which are however calculated at all frequencies using the approximation that $\operatorname{arctanh}\left(\sqrt{W(l)}\right)$ is Normal with variance $1/d$.

4 References

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

1: XG(NG) – ***real*** array *Input*

On entry: the NG univariate spectral estimates, $f_{xx}(\omega)$, for the x series.

2: YG(NG) – ***real*** array *Input*

On entry: the NG univariate spectral estimates, $f_{yy}(\omega)$, for the y series.

3: XYRG(NG) – ***real*** array *Input*

On entry: the real parts, $cf(\omega)$, of the NG bivariate spectral estimates for the x and y series. The x series leads the y series.

4: XYIG(NG) – ***real*** array *Input*

On entry: the imaginary parts, $qf(\omega)$, of the NG bivariate spectral estimates for the x and y series. The x series leads the y series.

Note: the two univariate and the bivariate spectra must each have been calculated using the same method of smoothing. For rectangular, Bartlett, Tukey or Parzen smoothing windows, the same cut-off point of lag window and the same frequency division of the spectral estimates must be used. For the trapezium frequency smoothing window, the frequency width and the shape of the window and the frequency division of the spectral estimates must be the same. The spectral estimates and statistics must also be unlogged.

5: NG – INTEGER *Input*

On entry: the number of spectral estimates in each of the arrays XG, YG, XYRG and XYIG. It is also the number of cross amplitude spectral and squared coherency estimates.

Constraint: NG ≥ 1.

6: STATS(4) – ***real*** array *Input*

On entry: the four associated statistics for the univariate spectral estimates for the x and y series. STATS(1) contains the degrees of freedom, STATS(2) and STATS(3) contain the lower and upper bound multiplying factors respectively and STATS(4) contains the bandwidth.

Constraints:

STATS(1) ≥ 3.0,
$0.0 <$ STATS(2) ≤ 1.0,
STATS(3) ≥ 1.0.

7: CA(NG) – ***real*** array *Output*

On exit: the NG cross amplitude spectral estimates $\hat{A}(\omega)$ at each frequency of ω.

8: CALW(NG) – ***real*** array *Output*

On exit: the NG lower bounds for the NG cross amplitude spectral estimates.

9: CAUP(NG) – ***real*** array *Output*

On exit: the NG upper bounds for the NG cross amplitude spectral estimates.

10: T – ***real*** *Output*

On exit: the critical value for the significance of the squared coherency, T.

11: SC(NG) – ***real*** array *Output*

On exit: the NG squared coherency estimates, $\hat{W}(\omega)$ at each frequency ω.

12: SCLW(NG) – ***real*** array *Output*

On exit: the NG lower bounds for the NG squared coherency estimates.

13: SCUP(NG) – ***real*** array *Output*

On exit: the NG upper bounds for the NG squared coherency estimates.

14: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, NG < 1,
or STATS(1) < 3.0,
or STATS(2) ≤ 0.0,
or STATS(2) > 1.0,
or STATS(3) < 1.0.

IFAIL $= 2$

A bivariate spectral estimate is zero. For this frequency the cross amplitude spectrum and squared coherency and their bounds are set to zero.

IFAIL $= 3$

A univariate spectral estimate is negative. For this frequency the cross amplitude spectrum and squared coherency and their bounds are set to zero.

IFAIL $= 4$

A univariate spectral estimate is zero. For this frequency the cross amplitude spectrum and squared coherency and their bounds are set to zero.

IFAIL = 5

A calculated value of the squared coherency exceeds 1.0. For this frequency the squared coherency is reset to 1.0 and this value for the squared coherency is used in the formulae for the calculation of bounds for both the cross amplitude spectrum and squared coherency. This has the consequence that both squared coherency bounds are 1.0.

If more than one failure of the types 2, 3, 4 and 5 occurs then the failure type which occurred at lowest frequency is returned in IFAIL. However the actions indicated above are also carried out for failures at higher frequencies.

7 Accuracy

All computations are very stable and yield good accuracy.

8 Further Comments

The time taken by the routine is approximately proportional to NG.

9 Example

The example program reads the set of univariate spectrum statistics, the two univariate spectra and the cross spectrum at a frequency division of $\frac{2\pi}{20}$ for a pair of time series. It calls G13CEF to calculate the cross amplitude spectrum and squared coherency and their bounds and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13CEF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NGMAX
      PARAMETER        (NGMAX=9)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             T
      INTEGER          I, IFAIL, J, NG
*     .. Local Arrays ..
      real             CA(NGMAX), CALW(NGMAX), CAUP(NGMAX), SC(NGMAX),
     +                 SCLW(NGMAX), SCUP(NGMAX), STATS(4), XG(NGMAX),
     +                 XYIG(NGMAX), XYRG(NGMAX), YG(NGMAX)
*     .. External Subroutines ..
      EXTERNAL         G13CEF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13CEF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NG
      READ (NIN,*) (STATS(I),I=1,4)
      READ (NIN,*) (XG(I),YG(I),XYRG(I),XYIG(I),I=1,NG)
      IFAIL = 1
*
      CALL G13CEF(XG,YG,XYRG,XYIG,NG,STATS,CA,CALW,CAUP,T,SC,SCLW,SCUP,
     +            IFAIL)
*
      WRITE (NOUT,*)
      IF (IFAIL.NE.0) THEN
         WRITE (NOUT,99999) 'G13CEF fails. IFAIL =', IFAIL
         WRITE (NOUT,*)
      END IF
      IF (IFAIL.NE.1) THEN
         WRITE (NOUT,*) '      Cross amplitude spectrum'
```

```
         WRITE (NOUT,*)
         WRITE (NOUT,*) '                    Lower     Upper'
         WRITE (NOUT,*) '          Value     bound     bound'
         DO 20 J = 1, NG
            WRITE (NOUT,99998) J - 1, CA(J), CALW(J), CAUP(J)
   20    CONTINUE
         WRITE (NOUT,*)
         WRITE (NOUT,99997) 'Squared coherency test statistic =', T
         WRITE (NOUT,*)
         WRITE (NOUT,*) '         Squared coherency'
         WRITE (NOUT,*)
         WRITE (NOUT,*) '                    Lower     Upper'
         WRITE (NOUT,*) '          Value     bound     bound'
         DO 40 J = 1, NG
            WRITE (NOUT,99998) J - 1, SC(J), SCLW(J), SCUP(J)
   40    CONTINUE
      END IF
      STOP
*
99999 FORMAT (1X,A,I3)
99998 FORMAT (1X,I5,3F10.4)
99997 FORMAT (1X,A,F12.4)
      END
```

9.2 Program Data

```
G13CEF Example Program Data
     9
 30.00000    .63858  1.78670    .33288
  2.03490 21.97712 -6.54995  0.00000
   .51554  3.29761    .34107 -1.19030
   .07640    .28782    .12335    .04087
   .01068    .02480  -.00514    .00842
   .00093    .00285  -.00033    .00032
   .00100    .00203  -.00039  -.00001
   .00076    .00125  -.00026    .00018
   .00037    .00107    .00011  -.00016
   .00021    .00191    .00007  0.00000
```

9.3 Program Results

```
 G13CEF Example Program Results

       Cross amplitude spectrum

                    Lower     Upper
          Value     bound     bound
     0    6.5499    3.9277   10.9228
     1    1.2382    0.7364    2.0820
     2    0.1299    0.0755    0.2236
     3    0.0099    0.0049    0.0197
     4    0.0005    0.0001    0.0017
     5    0.0004    0.0001    0.0015
     6    0.0003    0.0001    0.0010
     7    0.0002    0.0001    0.0007
     8    0.0001    0.0000    0.0018

 Squared coherency test statistic =      0.1926

          Squared coherency

                    Lower     Upper
          Value     bound     bound
     0    0.9593    0.9185    0.9799
     1    0.9018    0.8093    0.9507
     2    0.7679    0.5811    0.8790
     3    0.3674    0.1102    0.6177
     4    0.0797    0.0000    0.3253
     5    0.0750    0.0000    0.3182
     6    0.1053    0.0000    0.3610
```

```
     7    0.0952    0.0000    0.3475
     8    0.0122    0.0000    0.1912
```